Using inverse methods to recover basal velocities

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One of the biggest challenges for ice sheet models is a treatment of the basal boundary condition. Measurements at the base of the ice are useful to accurately describe and understand basal processes, but in the absence of a large coverage of such measurements, it is also necessary to derive basal conditions from data gathered at the surface or through remote sensing. This is, by necessity, an exercise in inverse modeling. We have developed an iterative method to find basal velocities and shear stresses, given data about ice thickness, rheological properties of the ice and surface velocities for a simplified model. The model we are considering is derived from a first order treatment of flow along a longitudinal cross section (Colinge and Rappaz, 1999). Incidentally, the same nonlinear Poisson equation also describes flow through a transverse cross section under the assumption of no out-of-plane gradients.

Our iterative scheme is derived from an idea developed by Kozlov and Maz'ya (1990). They suggested that the problem be solved by starting with an initial guess of basal boundary Neumann data (here essentially shear stress) and solve the forward problem with that and the measured surface Dirichlet data (surface velocity). This solution is then used to extract basal Dirichlet data (velocities) and a new model is solved with Dirichlet data at the bottom boundary and the measured Neumann data (zero shear stress) at the surface. This cycle can now be repeated. Kozlov and Maz'ya (1990) showed that this iteration converges in the linear case.

This method cannot be applied in a straight-forward manner to nonlinear ice flow. The reason is that convergence, even in the linear case, is extremely slow. We have developed an accelerated scheme and have shown that it works well in the case of flow through transverse cross sections of temperate valley glaciers (Maxwell et al., subm.). The method was applied to a glacier with measured basal velocities, and has been found to reproduce that well. It is also possible to qualitatively assess resolution, that is, to assess at what spatial scale one can expect to resolve variations in the basal velocity field.

Here we will apply the method by considering ice flowing along a flow line from slow inland ice to a fast moving ice stream. This is a challenging area for continental scale models, such as UAF's parallel ice sheet model (PISM, Bueler et al., ??, website), because it represents a switch between where ice is treated with the Shallow Ice Approximation to one where it is treated with the Shallow Shelf Approximation. We will discuss how well our inverse method can detect a hard switch from no-sliding to sliding conditions and we will then apply the method to data from the onset area of Bindschadler Ice Stream.

References:

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